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ON AVERSION TO POSITIVE RISKS AND PREFERENCE FOR NEGATIVE RISKS

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An intertemporal Von Neumann–Morgenstern utility function may account for aversion to risks when only gains are at stake and preference for risks when only losses are at stake. It may therefore be used to account for the findings of Markowitz and Kahneman and Tversky.

In a recent paper Kahneman and Tversky [K–T (1979)] reported the observation that at a wide range of wealth positions people exhibit both aversion to risk when only gains are at stake (positive risks) and preference for risk when only losses are at stake (negative risks). That is, when asked, most people preferred a gain of 3000 Israeli pounds (IL) with probability one to a gain of IL 4000 with probability 0.8. However, they preferred a loss of IL 4000 with probability 0.8 to a certain loss of IL 3000 [K–T (1979)]. To explain these findings K–T proposed an objective function which is defined on changes from a reference point rather than on final states and is concave for gains and convex for losses as illustrated in fig. 1 (K–T Figure 3). [1]

The observation of aversion to large positive risks and preference for large negative risks was made earlier by Markowitz (1952) who suggested an objective function which has three inflection points. Markowitz and K–T are thus in agreement with respect to large risks and I shall therefore limit the discussion to large risks.

A possible objection to these findings is their reliance on hypothetical choices rather than observed behavior. However, as K–T point out, all other methods that have been used to test utility theory also suffer from severe drawbacks. Here I accept the underlying assumption that people often know how they would behave in actual choice situations and do not disguise their true preferences.

The objective functions of Markowitz and K–T imply that tastes change with wealth. This is disturbing to economists who use the assumption of 'constant tastes' quite heavily. (It is hard to see how positive economics can do without this assumption and it is almost impossible to think of welfare economics without it.) Here I

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[1] K–T address other issues such as the Allais' (1953) paradox.

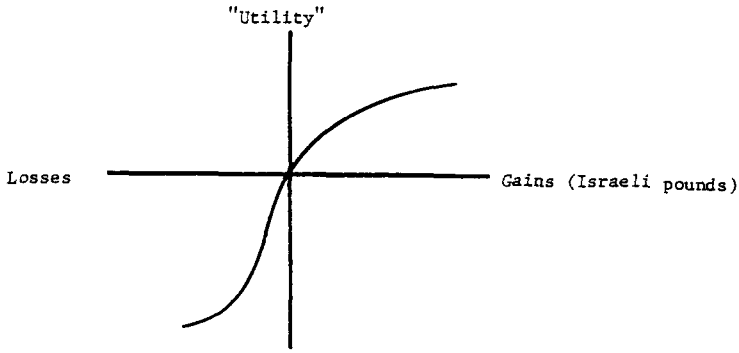

Fig. 1.

intend to show that the findings of Markowitz and K–T do not imply changes in tastes. I shall use the expected utility hypothesis to explain the attitude of preference for negative risks and aversion to positive risks. The elements used are: (a) a utility function which is defined on goods and only in an indirect way on wealth, and (b) the assumption that quantities of some goods are fixed due to past optimal and irreversible decisions. In particular I shall argue that if the consumer has chosen some goods on the basis of his initial wealth constraint w_i then his indirect utility of wealth, $V(w, w_i)$, may be concave for $w > w_i$ and convex for $w < w_i$.

I start with the simple case in which the consumer plans his consumption for two periods. Let c_t be consumption at period t, $(t = 1, 2)$ and assume that the consumer has a Von Neumann–Morgenstern utility function $U(c_1, c_2)$ which is differentiable and strictly quasi concave. For simplicity it is assumed that the consumer is free to lend and borrow at a zero rate of interest. Thus when his initial wealth is w he solves

$$\max_{c_1, c_2} U(c_1, c_2) \quad \text{subject to} \quad c_1 + c_2 = w. \tag{1}$$

Let $c_1(w)$, $c_2(w)$ be the solution to (1) and let

$$v(w) = U[c_1(w), c_2(w)] \tag{2}$$

be the maximum utility he can get from a given w, when he is free to choose both c_1 and c_2. Assume that starting from a given $w = w^*$, the consumer chose $c_1 = c_1(w^*)$ and after consuming it (i.e., between $t = 1$ and $t = 2$) he is informed that his wealth is w. The maximum utility he can get in this case is

$$V(w, w^*) = U[c_1(w^*), w - c_1(w^*)]. \tag{3}$$

$V()$ rather than $v()$ should be used for describing K–T findings, since they interviewed adults which have already consumed certain goods.

Note that $V()$ depends on the initial wealth (or K–T's reference point). However in this simple model it is difficult to generate a reasonable example in which $V()$ is concave for $w > w^*$ and convex for $w < w^*$. I shall therefore use a more elaborate version of the above model, in which there are two goods: a fixed durable good y and a variable good x. The two goods are aggregated into a consumption measure by

$$c_t = f(y, x_t), \qquad t = 1, 2, \tag{4}$$

where $f(\,)$ is a (production) function and the quantity of y is assumed to be the same for both periods. Thus once y is chosen at the beginning of the planning horizon the consumer cannot change its quantity. An example approximating such a good is the level of education one acquires in the early stages of life. [2] For simplicity let the prices of y and x be unity, then the consumer's problem at the beginning of the horizon when facing the wealth constraint $w = w^*$ is

$$\max_{y, x_1, x_2} \; U(c_1, c_2) \quad \text{subject to (4), and} \quad x_1 + x_2 + y = w^*. \tag{5}$$

Thus the consumer is viewed as using y and x as inputs for producing consumption. [3] Consider the case in which the consumer has no time preference and $f(\,)$ exhibits constant returns to scale. Under these assumptions $c_1(w^*) = c_2(w^*) = c(w^*)$, $x_1(w^*) = x_2(w^*) = x(w^*)$ and the optimum choice of inputs $y(w^*)$, $x(w^*)$ is that which minimizes the cost of producing $c(w^*)$. Under constant returns to scale the long-run average cost (LRAC) curve is horizontal and the short-run average cost (SRAC) curve is tangent to it as in fig. 2. The consumer's position at the beginning of the horizon is thus point A in fig. 2, which corresponds to point A in fig. 3. When $U(\,)$ is close to linear then $V(w, w^*)$ is similar to $f[y(w^*), x)$ and exhibits concavity for $w > w^*$ and convexity for $w < w^*$, provided that $|w - w^*|$ is not too small. In detail I assume that the short-run marginal cost (SRMC) curve, $1/f_2$, has a minimum near $c(w^*)$, [4] for all w^*. This implies that the marginal productivity of x, $f_2[y(w^*), x]$, increases at the beginning and starts to decline just before $x(w^*)$. It follows that approximately

$$f_{22}[y(w^*), x] \gtrless 0 \quad \text{as} \quad x \lesseqgtr x(w^*). \tag{6}$$

If after choosing $y(w^*)$, $x_1(w^*)$ the consumer is informed about a new wealth constraint w his utility will be

$$V(w, w^*) = U\{c_1(w^*), f[y(w^*), w - y(w^*) - x_1(w^*)]\}. \tag{7}$$

The second derivative of V, with respect to w, is given by

$$V_{11} = U_{22}f_2^2 + U_2 f_{22}\,. \tag{8}$$

[2] Other illiquid stocks such as a house, a car, or friends can be treated as fixed when small prospects are at stake, since in this case a fixed transaction cost makes the change in the stock prohibitively expensive.

[3] This approach introduced by Becker (1971) simplifies the exposition. Alternatively, a utility function defined on y, x_1, x_2 can be used.

[4] The subscript of f denotes partial derivative. Note that it is impossible to have the short-run marginal cost curve tangent to the long-run curve.

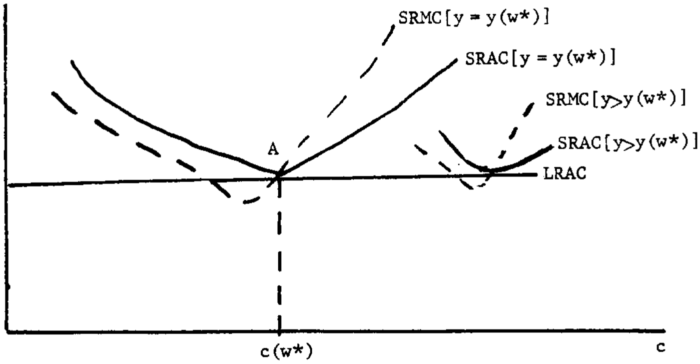

Fig. 2.

Since $U_2 > 0$, (6) implies that when $|U_{22}|$ is small we will have roughly

$$V_{11}(w, w^*) \lesseqgtr 0 \quad \text{as} \quad w \gtrless w^* . \tag{9}$$

Thus regardless of the initial w^*, the $V(\,)$ function is, in this case, concave for gains and convex for losses.

The assumption of constant returns to scale implies that the marginal product of the fixed factor becomes negative when the consumer is forced to a large reduction in the variable factor [see, for example, Ferguson (1972, p. 156, nt. 14)]. An example may be used to illustrate: consider a case in which the individual consumes only

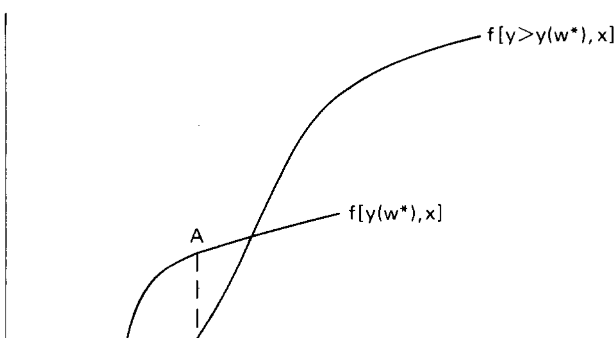

Fig. 3.

'transportation services' and uses a car as the fixed factor and gasoline as the variable factor of production. (Thus it is assumed that the transaction cost of selling the car and buying another one are prohibitive.) At the beginning of the horizon the consumer purchases a car which given the amount of money left for gasoline, produces 'transportation services' most efficiently. Assume now that in the second period the consumer is forced to reduce the amount of gasoline below the level that he initially planned. Wouldn't he be able to extract more 'transportation services' in the second period had he chosen a smaller and cheaper car (that is a lower level of the fixed factor)? Since the output 'transportation services' cannot be directly observed the reader should answer the question for himself. [5]

To sum up, I have argued that aversion to positive risks and preference for negative risks do not imply changes in taste (as a result of changes in wealth) and can be accounted for by a utility function which is defined on goods provided that the quantities of some goods are fixed due to past optimal and irreversible decisions. A somewhat similar approach is used in Eden (1977, 1979, forthcoming) to explain insurance without using the assumption of risk aversion and to account for the behavior of the insurance buying gambler.

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[5] Some insight can be gained by using the concept of 'human consumption capital' introduced by Friedman (the appendix to 1969) and Stigler and Becker (1977). In this context one can ask whether the capacity to enjoy sophisticated cultural activities reduces the ability to enjoy a 'lower level' and generally cheaper cultural activities. Does the refining of taste with respect to food reduce the ability to enjoy 'junk' food?